Mine Water and the Environment (2019) 38:81–92
https://doi.org/10.1007/s10230-018-00578-x

TECHNICAL ARTICLE

Analysis of Factors Influencing Floor Water Inrush in Coal Mines: A Nonlinear Fuzzy Interval Assessment Method

Xintong Wang[1] · Shucai Li[1] · Zhenhao Xu[1,2,3] · Peng Lin[1] · Jie Hu[1] · Wenyang Wang[1]

Received: 25 September 2017 / Accepted: 22 December 2018 / Published online: 3 January 2019

Abstract

A nonlinear fuzzy interval method for risk assessment of floor water inrush in coal mines was established, consisting of a multi-index evaluation system and a computational model. The multi-index evaluation system is formed by the destination, criteria, and indicator layers and the risk levels of floor water inrush were divided into five grades. Geological structure, hydrologic condition, the state of the floor aquifuge, and the mining condition were analyzed. Thirteen factors were considered as assessment indices. A computational model is proposed based on nonlinear fuzzy mathematics and the analytic hierarchical process (AHP). Considering the uncertainty of evaluation indices obtained from field exploration, the interval number was adopted to represent variables. Gaussian membership function was used to determine the membership function and membership degree, and the 1–9 AHP scales method was used to calculate the interval number judgment matrixes. The final evaluation levels were obtained by the relative superiority analysis method. The proposed method was successfully applied to the 13301 working face of the Wanglou mine and four additional coal mines in China. The results were highly consistent for these practical situations, which verify the reliability of this study.

Keywords Risk assessment · Fuzzy mathematics · Interval number · AHP · Weight

Introduction

The geological structure and hydrological conditions in coal mines can be quite complex. During the past 20 years, floor water inrush has affected coal mines in most of China's coal fields, including most of the late Paleozoic coal fields of north and south China and several Mesozoic and Cenozoic coal fields (Meng et al. 2012; Qiao et al. 2017; Qiu et al. 2017; Shi et al. 2014; Wang et al. 2018). Furthermore, the geological condition of mines has become more fragile, complex, and difficult to predict with the recent increase in mining depth, intensity, and large-scale exploitation (Qiao et al. 2018). Floor water inrush threatens coal mine safety, complicates tunnel construction, and causes tremendous loss of life and property during mining. Therefore, an accurate and efficient risk assessment of floor water inrush is essential.

A large quantity of research has been carried out to evaluate and predict the risk of floor water inrush. Wu et al. (2011) proposed the vulnerability index approach by coupling the analytic hierarchy process (AHP) and geographic information system (GIS). A secondary fuzzy comprehensive evaluation system was constructed to

✉ Zhenhao Xu
zhenhao_xu@sdu.edu.cn

Xintong Wang
wangxintong0722@163.com

Shucai Li
lishucai@sdu.edu.cn

Peng Lin
sddxytlp@163.com

Jie Hu
hjxz1996@163.com

Wenyang Wang
sduwwy@163.com

[1] Geotechnical and Structural Engineering Research Center, Shandong University, Jinan 250061, Shandong, China

[2] School of Qilu Transportation, Shandong University, Jinan 250061, Shandong, China

[3] Chinese Academy of Geological Sciences, Beijing 100037, China

evaluate the risk of floor water invasion in coal mines (Wang et al. 2012). Considering the lithology and structure features, a floor water inrush risk assessment method was put forward based on a conventional water inrush coefficient (Meng et al. 2012). Li et al. (2015) proposed an attribute synthetic evaluation system, which was combined with attribute mathematical theory and the analytic hierarchy process to evaluate the risk of floor water inrush. Grey relational analysis (GRA) and AHP were used to establish an evaluation model to reduce the uncertainty and reflect the importance of each water inrush index (Li and Chen 2016). Wu et al. (2017) presented a mathematical assessment system for water inrush risk in the coal floor based on the variable-weight model and unascertained measure theory. Based on fractal theory and an improved IAHP, a new evaluation model was developed to predict the potential for floor water inrush (Wang et al. 2017). Qiu et al. (2017) used an innovative combination of methods to assess floor water inrush risk based on the fuzzy Delphi AHP and GRA. A risk evaluation method was proposed based on principal component analysis, fuzzy mathematics, particle swarm optimization, and support vector classification, due to the limitations of the water inrush coefficient method (Shi et al. 2017). Yang et al. (2017) established a systematic method to evaluate the risk of water inrush through a coal seam floor using the geographic information system (GIS) and fuzzy set theory.

Each theory and method has its own advantages and uniqueness, but also some shortcomings. Floor water inrush in a coal mine is a complicated process with many influencing factors, but most studies have used simplified conditions and influencing factors. The quantitative relationship between water inrush and influence index cannot be deeply revealed. At the same time, the analysis of influencing factors is not comprehensive and cannot accurately reflect their influence. Moreover, the evaluation indices values and weight vectors are usually represented by constants in such studies. It is thus difficult to describe the uncertainty of influential factors and the actual situation cannot be fully explained by a geological survey.

The relationships among the various factors are highly nonlinear. The evaluation indexes obtained from field exploration are difficult to determine with constants and the judgment matrix formed by it should also be an interval. Meanwhile, it is difficult to determine the importance of an AHP evaluation index with constants, which also appears to have certain interval features. In the present study, a new floor water inrush assessment method for coal mines was established, using nonlinear fuzzy mathematics and AHP. An interval number was used that can clearly reflect the uncertainty and fuzziness of risk assessment. The values of evaluation indices, membership degrees, and weight vectors can be presented as interval numbers. Engineering practice was evaluated in the Wanglou mine and other four additional coal mines.

Multi Index Evaluation System of Floor Water Inrush

A new assessment model of floor water inrush in coal mines was established in the present study. The process of floor water inrush is extremely complex and its occurrence is affected by many factors. These factors can be divided into geological and hydrologic conditions, the state of the floor aquifuge, and mining conditions. A multi-index evaluation system was established consisting of three hierarchies (Fig. 1). The first hierarchy is the destination layer: risk assessment of floor water inrush for coal mines (A). The second hierarchy is the criteria layer, and has four parts: geological structure index (B1), hydrogeology index (B2), floor aquifuge index (B3), and mining condition index (B4) (Li and Chen 2016). The third hierarchy, the indicator layer, is made up of thirteen factors: fault density (C1), fault -water transmitting ability (C2), the development degree of fractures and faults (C3), hydraulic pressure of the confined aquifer (C4), water supply condition and aquifer water yield property (C5), karst development (C6), floor aquifuge thickness (C7), floor aquifuge strength (C8), key aquifuge of floor (C9), mining thickness (C10), mining depth (C11), and inclined length of the mining face (C12) (Wang et al. 2012).

The risk of a floor water inrush can be classified into five grades: I (very high), II (high), III (medium), IV (low), and V (very low) based on previous studies (Li et al. 2015; Li and Chen 2016; Qiu et al. 2017; Yang et al. 2017). The specific risk standards are presented in Table 1, based on the proposed index system and previous research (Wang et al. 2012).

Geological Structure Index (B1)

Fractures and faults are the main geological structural index in the mine floor. A fault is a fissure-concentrated area that shortens the distance between the floor and the aquifer. The spatial relation between the mine floor and aquifer can be changed by the dislocation of faults, which reduces the thickness of the impermeable layer and the strength of the aquiclude near the fault, meaning that a floor water inrush is more likely to occur. Moreover, the integrity of the rock mass structure may be destroyed. In this study, the geological structure index was divided into three indices: fault density (C1), fault-water transmitting ability (C2), and the development degree of fractures and faults (C3).

1. Fault density: Fault density is defined as the ratio of the fracture volume to the total rock mass volume and

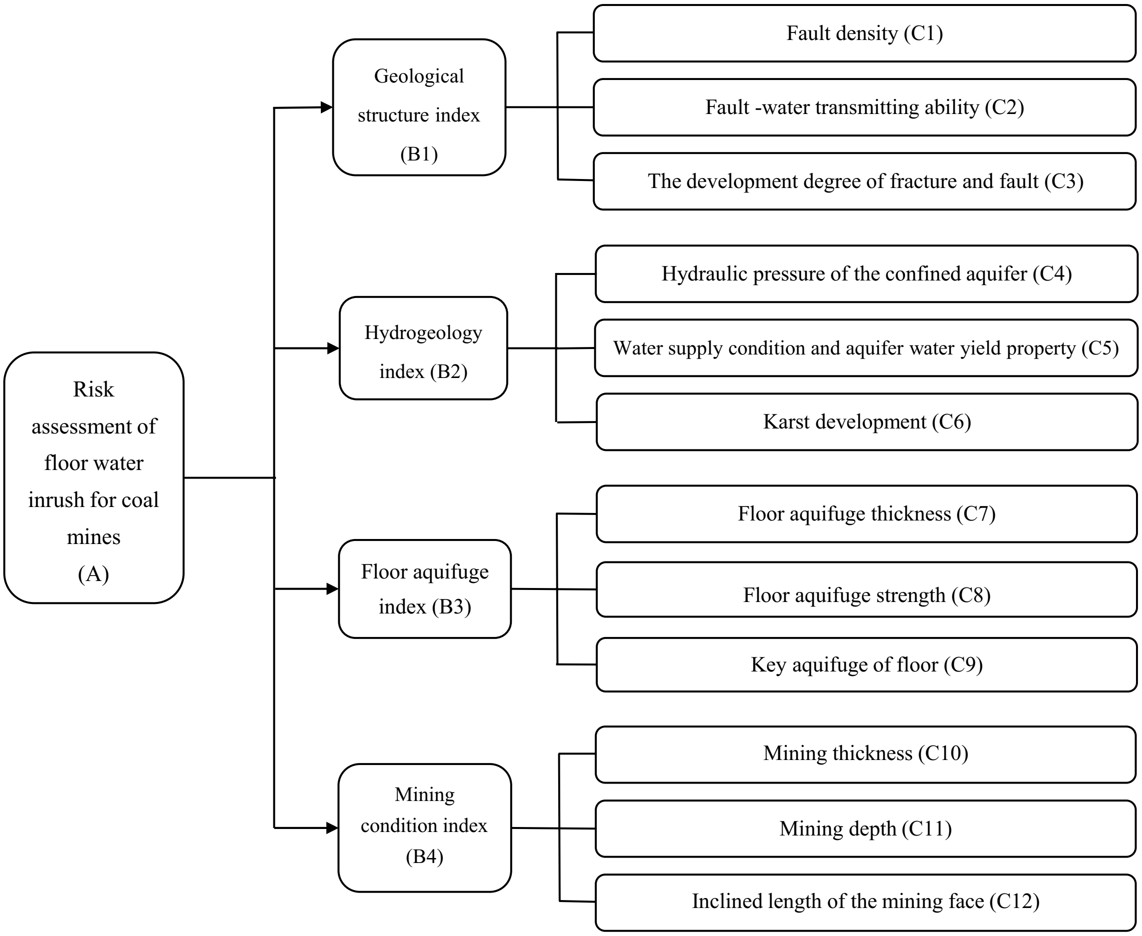

Fig. 1 Multi-index evaluation indices of risk assessment of floor water inrush

Table 1 Grade division of evaluation indices of floor water inrush

Evaluation indices	Grade division				
	I	II	III	IV	V
Fault density (C1) (lip/km^2)	> 2.9	[2.5, 2.9]	[2.1, 2.5]	[1.5, 2.1]	< 1.5
Fault water transmitting ability (C2)	[8, 10]	[6, 8]	[4, 6]	[2, 4]	[0, 2]
The development degree of fracture and fault (C3)	[8, 10]	[6, 8]	[4, 6]	[2, 4]	[0, 2]
Hydraulic pressure of the confined aquifer (C4) (MPa)	> 4	[3, 4]	[2, 3]	[1, 2]	< 1
Water supply condition and water yield property of aquifer (C5)	[8, 10]	[6, 8]	[4, 6]	[2, 4]	[0, 2]
Karst development degree (C6)	[8, 10]	[6, 8]	[4, 6]	[2, 4]	[0, 2]
Floor aquifuge thickness (C7) (m)	< 25	[25, 50]	[50, 75]	[75, 100]	> 100
Floor aquifuge strength (C8) (MPa)	< 1.2	[1.2, 1.7]	[1.7, 2.1]	[2.1, 2.6]	> 2.6
Key aquifuge of floor (C9)	[0, 2]	[2, 4]	[4, 6]	[6, 8]	[8, 10]
Mining thickness (C10) (m)	> 2.5	[1.6, 2.5]	[1.4, 1.6]	[1.2, 1.4]	< 1.2
Mining depth (C11) (m)	> 800	[650, 800]	[550, 650]	[400, 550]	< 400
Inclined length of the mining face (C12) (m)	> 178	[132, 178]	[88, 132]	[42, 88]	< 42

includes the effects of damage to the mine floor and surrounding rock caused by the fault. The greater the fault density is, the greater the damage to the coal seam and the surrounding rock. The regional tectonic stress field and the physical and mechanical properties of the rock greatly influence the fault density. Fault density maps and rosette plots can be plotted according to the number distribution density of faults and trace length density in different regions.

2. Fault-water transmitting ability: The ability to transmit groundwater through fissures, which is related to the connectivity and permeability of fractures, is directly related to floor water inrush events. The stronger the transmission capacity, the greater the probability of disaster. Specifically, the distribution density and connectivity rate of pinch-out and crossings affect fracture connectivity. Aquifer permeability often depends more on the amount of fissures and their characteristics than porous media flow aspects, such as particle diameter. Fault-water transmitting ability is quantitatively graded according to five grades (Table 1).
3. The development degree of fractures and faults: Highly-developed fractures and faults can provide more water storage space and water conductive channels. The development of fractures and faults refers to their size and degree of in-filling. The size of fractures and faults includes the depth of the cut, their length, and their number (Table 2). The greater the fault length, the more fracture development, and the more prone the area is to floor water inrush. On the other hand, faults can be destroyed by different degrees of mechanics during their formation, having either a loose or secondary fracture zone, as well as by in-filling. The lithology of the fractures and faults directly determines and controls the width of the fault zone and the extent of fracturing and fracture development.

Hydrogeology Index (B2)

The confined floor aquifer is the most important water-filling source for most floor water inrush events in coal mines. The hydraulic pressure of the confined aquifer, water supply, aquifer water yield property, and karst development affect floor water inrush.

1. Hydraulic pressure of the confined aquifer: The hydraulic pressure of the karst aquifer is the driving force of water inrush from the floor. A floor water inrush will occur when the free water level of the confined aquifer is above the elevation of the coal mine floor. Moreover, the confined aquifer should also have adequate head pressure. The confined aquifer and fractures are the main water passageways. Water inrush from the floor cannot occur until the hydraulic pressure is large enough, unless the excavation exposes these main water passageways directly. The hydraulic pressure of the confined aquifer affects the inrush mainly through the influence of hydrodynamic and hydrostatic pressures on the aquifuge.
2. Water supply condition and water yield properties of aquifer: The hydrogeological conditions and structural conditions of a mining area greatly influence the water supply condition of aquifer, and the aquifer's water yield property is the material foundation of a floor water inrush. Specifically, the water supply condition refers to the water yield of the aquifer, and the elevation, water temperature, and chemical properties of the groundwater. The amount of inrush is determined by the degree of water abundance and the recharge conditions of the aquifer. Moreover, the greater the permeability, the stronger its water conductivity, and the greater the threat of an inrush. The water yield is related to fissure development, lithology, and rock thickness. In the present study, the water supply condition and yield properties of the aquifer is divided into

Table 2 A detailed description for the development degree of fracture and fault (C3)

Grade division	Description	The development degree of fracture and fault	
I	Extremely strong developed fracture and fault	There are more than 3 groups of messy fissures, mainly weathered and tectonic. The fracture spacing is less than 0.2 m, mainly with open cracks, and generally have fillers. The crack width is greater than 5 mm. Almost all rock masses are cut into gravel	[8, 10]
II	Strong developed fracture and fault	There are more than 3 groups of messy fissures, which are mainly weathered and tectonic. Most of the fracture spacing is less than 0.4 m, and generally have fillers. The crack width is about 3–5 mm. Almost all rock masses are cut into small pieces	[6, 8]
III	Moderately developed fracture and fault	There are 2–3 groups of fissures. Most of the fracture spacing is less than 0.4 m with few fillers. The crack width is about 2–3 mm. Almost all rock masses are cut into big pieces	[4, 6]
IV	Weakly developed fracture and fault	There are 1–2 groups of regular fissures. Most of the fracture spacing is less than 1 m. The crack width is about 1–2 mm	[2, 4]
V	Undeveloped fracture and fault	There are almost no cracks, and the rare remaining cracks are less than 1 mm wide	[0, 2]

five grades, which is quantitatively classified by expert scoring.

3. Karst development: Karst development has obvious heterogeneity and anisotropy. This results in a large difference in the water abundance of an aquifer in horizontal and vertical directions. In the horizontal direction, the water abundance is strong and dynamic reserves are abundant in a karst aquifer. In the vertical direction, the aquifer is rich in water where the karst is well developed. However, water abundance and permeability will decrease with buried depth.

Floor Aquifuge Index (B3)

1. Floor aquifuge thickness: The thickness of an aquifuge is the distance from the mine floor to the top of the aquifuge. When the total thickness of the aquifuge increases, its ability to resist hydraulic damage increases, and the smaller the probability of an inrush.
2. Floor aquifuge strength: The ability of various lithologies and different lithological combinations to resist water pressure and the water resistance of an aquifuge can be very different. Moreover, under the same conditions, if the mechanical strength of a sandstone or carbonate layer is strong, then the ability to resist water inrush is strong. In contrast, the mechanical strength of a plastic shale layer is poor, making the possibility of an inrush greater.
3. Key aquifuge of floor: The key strata refer to the rock layers with the highest strength in the floor. The location of key strata in the floor aquifuge determines its function. If the key strata are at the bottom of the aquifuge, then a fracture that penetrates it can lead to an inrush. If the key strata are at the top of the aquifuge, the key rock formation will be destroyed during mining, which will lead to an inrush. If the key strata are in the middle of the aquifuge, the key stratum can play a great role in resisting an inrush.

Mining Condition Index (B4)

An increase in the mining thickness and depth, or the inclined length of the mine face will change the mine pressure, potentially inducing an inrush.

1. Mining thickness and depth: The mining thickness and depth are the dominant factors of deformation and floor failure. With an increased mining depth, the original stress of the rock and mine pressure increases, which can cause geological disasters to occur.
2. Inclined length of the mining face: When the inclined length of the mining face is long, the mined out area will be large. Then, floor failure can more easily occur due to the rock pressure. If geological structure exists in that areas, the damage to the mine floor will be more serious and an inrush is more likely.

Thus, a floor water inrush results from multiple factors. When analyzing the risk of an inrush, these factors should be assigned different weights, so that a scientific and reasonable risk evaluation can be made.

Computational Model

The occurrence and development of a floor water inrush in coal mines is a complicated process with many influential factors and strong randomness. The relationship among the various factors is highly nonlinear. We have established a computational model based on nonlinear fuzzy mathematics and AHP. The interval membership function and factor weights were quantified and a relative superiority analysis of interval matrix was carried out. A new risk assessment computational model of floor water inrush has been established (Moore and Lodwick 2003).

The Nonlinear Fuzzy Evaluation Function

Generally, the evaluation indices obtained from field exploration are difficult to determine with fixed values. Therefore, an interval number is used to reflect the uncertainty and fuzziness. In the present study, result vectors, weight vectors, and judgment matrixes were all expressed as interval numbers. The final result vector can be derived from the comprehensive evaluation of the nonlinear fuzzy evaluation functions, as shown below:

$$N_A = W_B \cdot \left[N_{B1}, N_{B2}, N_{B3}, N_{B4}\right]^T \tag{1}$$

$$N_{Bi} = W_i \cdot N_C \tag{2}$$

In which,

$$N_A = \left[\left[\underline{s_1}, \overline{s_1}\right], \left[\underline{s_2}, \overline{s_2}\right], \ldots, \left[\underline{s_4}, \overline{s_4}\right]\right] \tag{3}$$

$$W_B = \left[\left[\underline{w_{B1}}, \overline{w_{B1}}\right], \left[\underline{w_{B2}}, \overline{w_{B2}}\right], \left[\underline{w_{B3}}, \overline{w_{B3}}\right], \left[\underline{w_{B4}}, \overline{w_{B4}}\right]\right] \tag{4}$$

$$P_C = \begin{bmatrix} \begin{bmatrix} \underline{P_{i1\mathrm{I}}}, \overline{P_{i1\mathrm{I}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i1\mathrm{II}}}, \overline{P_{i1\mathrm{II}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i1\mathrm{III}}}, \overline{P_{i1\mathrm{III}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i1\mathrm{IV}}}, \overline{P_{i1\mathrm{IV}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i1\mathrm{V}}}, \overline{P_{i1\mathrm{V}}} \end{bmatrix} \\ \begin{bmatrix} \underline{P_{i2\mathrm{I}}}, \overline{P_{i2\mathrm{I}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i2\mathrm{II}}}, \overline{P_{i2\mathrm{II}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i2\mathrm{III}}}, \overline{P_{i2\mathrm{III}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i2\mathrm{IV}}}, \overline{P_{i2\mathrm{IV}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{i2\mathrm{V}}}, \overline{P_{i2\mathrm{V}}} \end{bmatrix} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \begin{bmatrix} \underline{P_{ij\mathrm{I}}}, \overline{P_{ij\mathrm{I}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{ij\mathrm{II}}}, \overline{P_{ij\mathrm{II}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{ij\mathrm{IV}}}, \overline{P_{ij\mathrm{IV}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{ij\mathrm{IV}}}, \overline{P_{ij\mathrm{IV}}} \end{bmatrix} & \begin{bmatrix} \underline{P_{ij\mathrm{V}}}, \overline{P_{ij\mathrm{V}}} \end{bmatrix} \end{bmatrix} \quad (5)$$

where N_A and N_{Bi} are the result vector of the destination layer (A) and the criteria layer (B), respectively. W_B is the weights of the criteria layer (B). N_C is the judgment matrix of the indicator layer (C). W_i is the weight vector of the indicator layer (C) determined by analytic hierarchy process. P_{ijk} is the membership degree for index x_{ij} belonging to risk grade k, $i=1$–4, $j=1$–13, and $k=I$–IV.

If the interval number are $\left[\underline{x}, \overline{x}\right]$ and $\left[\underline{y}, \overline{y}\right]$, the arithmetic of the interval number can be defined as (Sun and Wu 2014):

$$\left[\underline{x}, \overline{x}\right] \pm \left[\underline{y}, \overline{y}\right] = \left[\underline{x} \pm \underline{y}, \overline{x} \pm \overline{y}\right] \quad (6)$$

$$\left[\underline{x}, \overline{x}\right] \times \left[\underline{y}, \overline{y}\right] = \left[\min\{\underline{x}\cdot\underline{y}, \underline{x}\cdot\overline{y}, \overline{x}\cdot\underline{y}, \overline{x}\cdot\overline{y}\}, \max\{\underline{x}\cdot\underline{y}, \underline{x}\cdot\overline{y}, \overline{x}\cdot\underline{y}, \overline{x}\cdot\overline{y}\}\right] \quad (7)$$

$$\left[\underline{x}, \overline{x}\right] \div \left[\underline{y}, \overline{y}\right] = \left[\min\left\{\frac{\underline{x}}{\underline{y}}, \frac{\underline{x}}{\overline{y}}, \frac{\overline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}\right\}, \max\left\{\frac{\underline{x}}{\underline{y}}, \frac{\underline{x}}{\overline{y}}, \frac{\overline{x}}{\underline{y}}, \frac{\overline{x}}{\overline{y}}\right\}\right] \quad (8)$$

$$a \times \left[\underline{x}, \overline{x}\right] = \begin{cases} a \times \left[\underline{x}, \overline{x}\right] & a > 0 \\ a \times \left[\overline{x}, \underline{x}\right] & a < 0. \end{cases} \quad (9)$$

Membership Degree Based on Gaussian Function

We used the membership function to calculate the membership degree. There are many kinds of membership functions, including normal type, triangular fuzzy numbers, trapezoidal, and ridge membership. Despite the different forms of membership functions, the final analysis results are almost consistent. Therefore, the selection of the membership function has little effect on the conclusions. In this paper, the Gaussian membership function (Moore and Lodwick 2003) was used. Gaussian membership function is defined by a central value x_0 and a standard deviation d ($k>0$). The membership degree of the evaluation index can be calculated by the different forms of Gaussian membership function in the following forms.

The quantitative index and qualitative index are two kinds of evaluation indices of floor water inrush. For quantitative index, when evaluation index value belongs to the minimum or maximum risk level, the membership function can be adopted as follow.

$$P_{ijk}(x) = \begin{cases} 1 & x \le a \\ e^{-\left[\frac{(x-x_0)}{d}\right]^2} & x > a \end{cases} \quad (10)$$

$$P_{ijk}(x) = \begin{cases} e^{-\left[\frac{(x-x_0)}{d}\right]^2} & x \le a \\ 1 & x > a. \end{cases} \quad (11)$$

If the value of the evaluation index is mid-level, the membership function can be calculated in the following forms.

$$P_{ijk}(x) = e^{-\left[\frac{(x-x_0)}{d}\right]^2} \quad (12)$$

In the formula, $a_0 = (\overline{a} + \underline{a})/2$, $\overline{a}$ and $\underline{a}$ are the boundaries of the evaluation index grade. If the index value is the boundary of two levels, the membership degree corresponding to two grades should all be 0.5:

$$P = e^{-\left[\frac{(\overline{a}-\underline{a})}{2d}\right]^2} = 0.5 \quad (13)$$

$$d = (\overline{a} - \underline{a})/1.67. \quad (14)$$

Moreover, if $\sum_{k=1}^{5} P_{ijk} \neq 1$, the membership degrees should be normalized.

The assessment indices of floor water inrush obtained from engineering exploration are, in fact, mostly descriptive, for instance, fault water transmitting ability, karst development, and the water supply condition and water yield property of the aquifer. For calculation convenience, these qualitative indices should be quantified. We assumed that the total range of risk grade for qualitative indices is [0, 10]. The width of each range is 2.

Based on geological structure B1, hydrogeology index B2, floor aquifuge index B3 and mining condition index B4, the membership functions of evaluation index C1–C12 corresponding to every grade in evaluation risk of floor water inrush are derived, shown as Table 3 and Fig. 2.

Interval Weight Calculation

The traditional AHP usually adopts a constant value to describe the importance of evaluation indices. In this study, interval numbers were used to address the uncertainty of engineering and determine the weight vectors. The interval number judgment matrixes of all indices can be calculated

Table 3 The membership functions of evaluation index C1–C12

Indices	Grade division				
	I	II	III	IV	V
C1	$P_I(x)=\begin{cases}1 & x>3.2\\ e^{-\left[\frac{(x-3.2)}{0.36}\right]^2} & x\le 3.2\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-2.7)}{0.24}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-2.3)}{0.24}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-1.8)}{0.36}\right]^2}$	$P_V(x)=\begin{cases}1 & x\le 0.75\\ e^{-\left[\frac{(x-0.75)}{0.9}\right]^2} & x>0.75\end{cases}$
C4	$P_I(x)=\begin{cases}1 & x>4.5\\ e^{-\left[\frac{(x-4.5)}{0.6}\right]^2} & x\le 4.5\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-3.5)}{0.6}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-2.5)}{0.6}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-1.5)}{0.6}\right]^2}$	$P_V(x)=\begin{cases}1 & x\le 0.5\\ e^{-\left[\frac{(x-0.5)}{0.6}\right]^2} & x>0.5\end{cases}$
C7	$P_I(x)=\begin{cases}1 & x<12.5\\ e^{-\left[\frac{(x-12.5)}{15}\right]^2} & x\ge 12.5\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-37.5)}{15}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-62.5)}{15}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-87.5)}{15}\right]^2}$	$P_V(x)=\begin{cases}1 & x\ge 112.5\\ e^{-\left[\frac{(x-112.5)}{15}\right]^2} & x<112.5\end{cases}$
C8	$P_I(x)=\begin{cases}1 & x\le 0.6\\ e^{-\left[\frac{(x-0.6)}{0.72}\right]^2} & x>0.6\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-1.45)}{0.3}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-1.9)}{0.24}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-2.35)}{0.3}\right]^2}$	$P_V(x)=\begin{cases}1 & x>3.3\\ e^{-\left[\frac{(x-3.3)}{0.8}\right]^2} & x\le 3.3\end{cases}$
C10	$P_I(x)=\begin{cases}1 & x>3\\ e^{-\left[\frac{(x-3)}{0.6}\right]^2} & x\le 3\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-2.05)}{0.54}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-1.5)}{0.12}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-1.3)}{0.12}\right]^2}$	$P_V(x)=\begin{cases}1 & x\le 0.6\\ e^{-\left[\frac{(x-0.6)}{0.72}\right]^2} & x>0.6\end{cases}$
C11	$P_I(x)=\begin{cases}1 & x>1000\\ e^{-\left[\frac{(x-1000)}{240}\right]^2} & x\le 1000\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-725)}{90}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-600)}{60}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-475)}{90}\right]^2}$	$P_V(x)=\begin{cases}1 & x\le 200\\ e^{-\left[\frac{(x-200)}{240}\right]^2} & x>200\end{cases}$
C12	$P_I(x)=\begin{cases}1 & x>199\\ e^{-\left[\frac{(x-199)}{25}\right]^2} & x\le 199\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-155)}{27.5}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-110)}{26.35}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-65)}{27.5}\right]^2}$	$P_V(x)=\begin{cases}1 & x\le 21\\ e^{-\left[\frac{(x-21)}{25}\right]^2} & x>21\end{cases}$
Qualitative indices (C2, C3,C5, C6)	$P_I(x)=\begin{cases}1 & x>9\\ e^{-\left[\frac{(x-9)}{1.2}\right]^2} & x\le 9\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-7)}{1.2}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-5)}{1.2}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-3)}{1.2}\right]^2}$	$P_V(x)=\begin{cases}1 & x\le 1\\ e^{-\left[\frac{(x-1)}{1.2}\right]^2} & x>1\end{cases}$
Qualitative indices (C9)	$P_I(x)=\begin{cases}1 & x\le 1\\ e^{-\left[\frac{(x-1)}{1.2}\right]^2} & x>1\end{cases}$	$P_{II}(x)=e^{-\left[\frac{(x-3)}{1.2}\right]^2}$	$P_{III}(x)=e^{-\left[\frac{(x-5)}{1.2}\right]^2}$	$P_{IV}(x)=e^{-\left[\frac{(x-7)}{1.2}\right]^2}$	$P_V(x)=\begin{cases}1 & x\le 9\\ e^{-\left[\frac{(x-9)}{1.2}\right]^2} & x>9\end{cases}$

The evaluation index are: fault density (C1), fault -water transmitting ability (C2), the development degree of fracture and fault (C3), hydraulic pressure of the confined aquifer (C4), water supply condition and aquifer water yield property (C5), karst development (C6), floor aquifuge thickness (C7), floor aquifuge strength (C8), key aquifuge of floor (C9), mining thickness (C10), mining depth (C11) and inclined length of the mining face (C12)

by using the 1–9 scales method (Saaty 1990), to obtain the weight vectors of the interval numbers. The judgment matrix V_i of the indices and the weight vector W can be constructed in the following forms.

$$V_i = \left[\left[\underline{v_{ij}}, \overline{v_{ij}}\right]\right]_{n\times n} \tag{15}$$

$$w_{ij} = \frac{\left(\prod_{j=1}^{n} v_{ij}\right)^{1/n}}{\sum_{i=1}^{n}\left(\prod_{j=1}^{n} v_{ij}\right)^{1/n}} \tag{16}$$

$$W = \left[w_{ij}\right]_{1\times n} = \left[\left[k\underline{w_{ij}}, m\overline{w_{ij}}\right]\right]_{1\times n} \tag{17}$$

$$\text{in which, } v_{ij} = \frac{1}{v_{ji}} \tag{18}$$

$$v_{ij}v_{jk} = v_{jj}v_{ik} \tag{19}$$

$$k = \sqrt{\sum_{j=1}^{n}\left(1/\sum_{i=1}^{n}\overline{v_{ij}}\right)} \tag{20}$$

$$m = \sqrt{\sum_{j=1}^{n}\left(1/\sum_{i=1}^{n}\underline{v_{ij}}\right)} \tag{21}$$

where n is the number of evaluation indices of the criteria layer (B) and j = 1–n. V_i is a consistent matrix of the interval number. The weights of all indices can thus be determined. Then, the result vectors can be obtained using the nonlinear fuzzy evaluation functions.

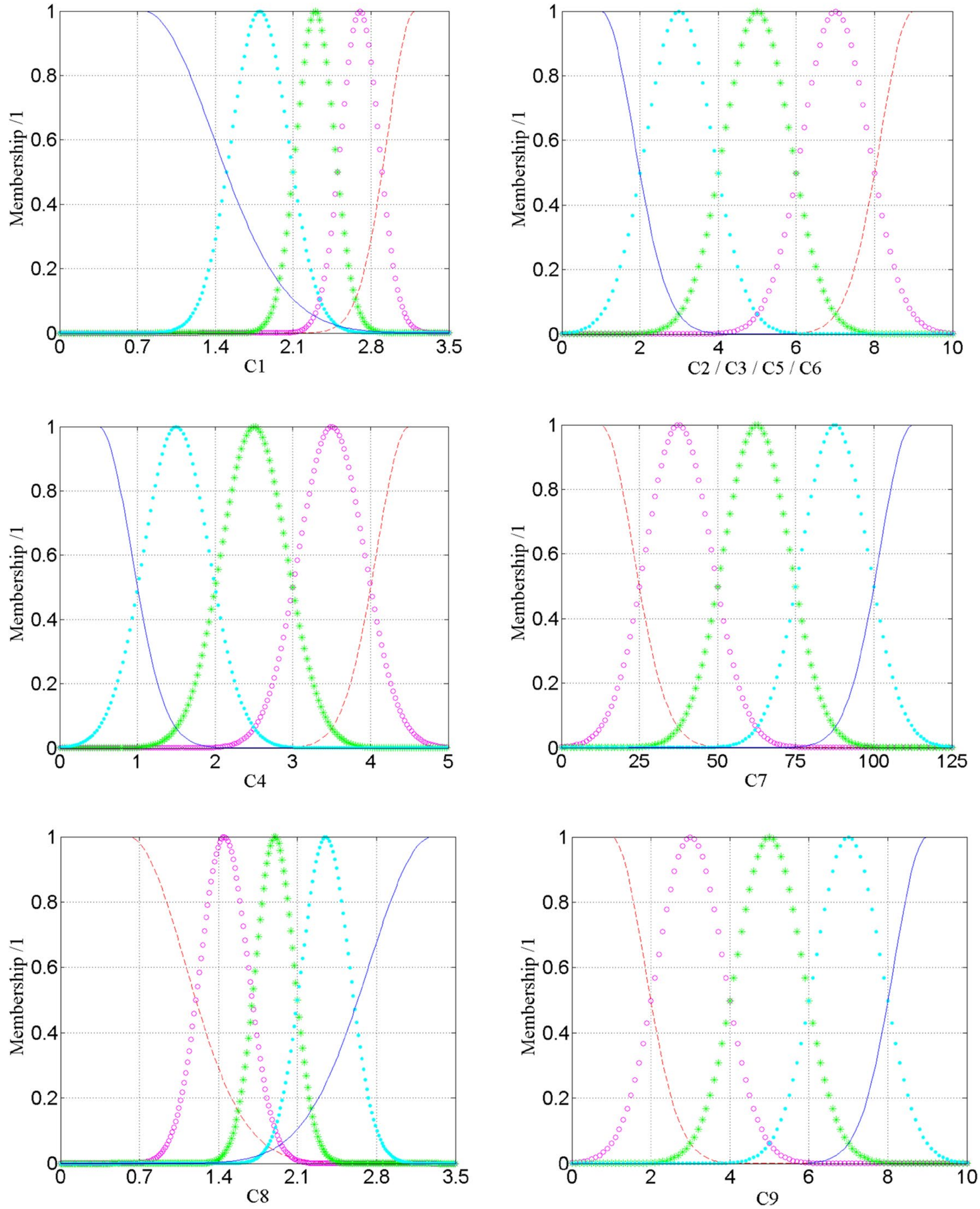

Fig. 2 The membership functions of different evaluation indices

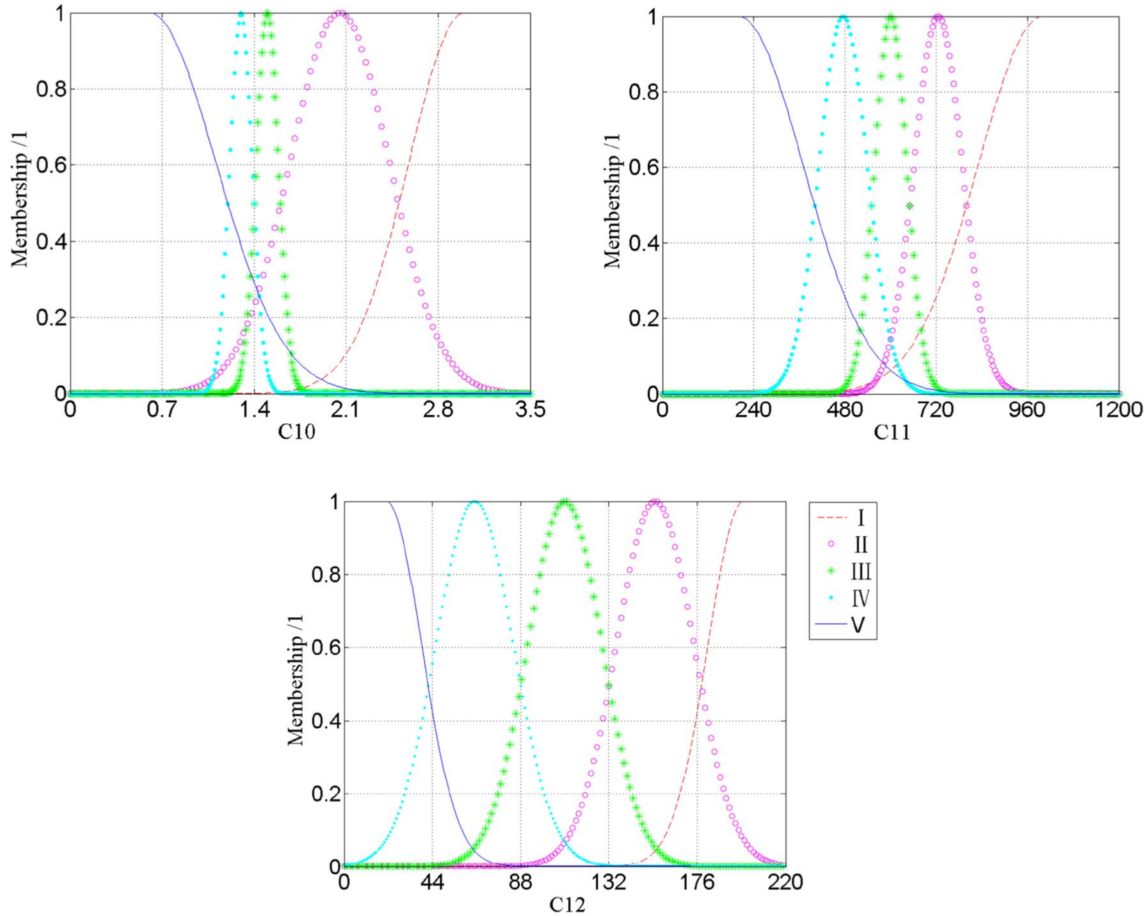

Fig. 2 (continued)

Relative Superiority Analysis

The final evaluation level is calculated by the relative superiority analysis method. If $A = \left[\underline{a}, \overline{a}\right]$ and $B = \left[\underline{b}, \overline{b}\right]$ are two of the interval number vectors for risk grade evaluation, the relative dominance can be defined as follows:

$$(A > B) = \begin{cases} 1 & a^- \geq b^+ \\ 1 - \frac{(b^+ - a^-)^2}{2l(A)l(B)} & b^- \leq a^- \leq b^+ \leq a^+ \\ \frac{a^+ + a^- - 2b^-}{2l(B)} & b^- \leq a^- \leq a^+ \leq b^+ \\ \frac{2a^+ - b^+ - b^-}{2l(A)} & a^- \leq b^- \leq b^+ \leq a^+ \\ \frac{(a^+ - b^-)^2}{2l(A)l(B)} & a^- \leq b^- \leq a^+ \leq b^+ \\ 0 & a^+ \leq b^-. \end{cases} \tag{22}$$

In the formula, $l(A) = a^+ - a^-$ and $l(B) = b^+ - b^-$. The matrix $S = \left[s_{ij}\right]_{5\times5}$ of relative dominance can be sorted by the ranking method for a fuzzy complementary matrix. The weight of the relative dominance matrix is obtained as follows:

$$P_i = \frac{\left(\sum_{j=1}^{n} s_{ij} + \frac{n}{2} - 1\right)}{n(n-1)}. \tag{23}$$

The grade for the maximum weight is the final risk level of floor water inrush.

Engineering Application

Nonlinear Fuzzy Floor Water Inrush Evaluation of the Wanglou Mine

In the present study, the 13301 working face of the Wanglou mine (Gao 2014; Xiao 2014) in Jining, Shandong Province, was selected as the study area. The design capacity of the mine is 900 thousand tons and the 13301 working face is the main face of the no. 3 mining area. The strike longwall retreating method is used there. The working face is 1160 m

Table 4 Values of evaluation indices of floor water inrush

Assessment index	13301 working face in Wanglou Mine	3612 working face in Huaibei Permian mining area	3137 working face in Zhaogezhuang Mine	7304 working face in Taoyang Mine	31401 working face in Qilianta Mine
C1	[2.8, 3.3]	[3.1, 3.9]	[2.3, 2.8]	[3.2, 3.7]	[1.8, 2.3]
C2	[6, 8]	[8, 10]	[4, 6]	[6, 8]	[6, 8]
C3	[8, 10]	[8, 10]	[4, 6]	[4, 6]	[4, 6]
C4	[1.81, 3.82]	[1.5, 4.2]	[4.6, 6.76]	[0.6, 1.1]	[2.1, 2.8]
C5	[6, 8]	[6, 8]	[6, 8]	[4, 8]	[4, 8]
C6	[4, 6]	[6, 8]	[2, 4]	[2, 4]	[6, 8]
C7	[65, 215]	[39, 64]	[105, 140]	[19, 56]	[120, 190]
C8	[1.6, 2.24]	[1.2, 1.8]	[1.8, 3.2]	[1.1, 4.2]	[1.5, 1.8]
C9	[8, 10]	[4, 6]	[6, 8]	[6, 8]	[4, 6]
C10	[1.9, 2.4]	[2.6, 2.88]	[8.5, 11.2]	[1.05, 1.34]	[1.8, 2.4]
C11	[900, 960]	[150, 420]	[1030, 1140]	[390, 440]	[180, 250]
C12	[150, 168]	[120, 160]	[20, 120]	[84, 92]	[120, 150]

The evaluation index are: fault density (C1), fault -water transmitting ability (C2), the development degree of fracture and fault (C3), hydraulic pressure of the confined aquifer (C4), water supply condition and aquifer water yield property (C5), karst development (C6), floor aquifuge thickness (C7), floor aquifuge strength (C8), key aquifuge of floor (C9), mining thickness (C10), mining depth (C11) and inclined length of the mining face (C12)

Table 5 Interval membership degrees

Assessment index	Grade division				
	I	II	III	IV	V
C1	[0.29, 1]	[0.02, 0.84]	0	0	0
C2	[0, 0.5]	[0.5, 1]	[0, 0.5]	0	0
C3	[0.5, 1]	[0, 0.5]	0	0	0
C4	[0, 0.26]	[0, 0.76]	[0.25, 1]	[0, 0.75]	0
C5	[0, 0.5]	[0.5, 1]	[0, 0.5]	0	0
C6	0	[0, 0.5]	[0.5, 1]	[0, 0.5]	0
C7	0	[0, 0.35]	[0, 0.97]	[0.1, 1]	[0, 1]
C8	[0, 0.14]	[0, 0.77]	[0.13, 1]	[0, 0.87]	0
C9	0	0	0	[0, 0.5]	[0.5, 1]
C10	[0.03, 0.36]	[0.65, 0.92]	0	0	0
C11	[0.84, 0.97]	[0, 0.02]	0	0	0
C12	[0.02, 0.21]	[0.80, 0.96]	[0, 0.09]	0	0

long. The distance between the coal seam roof and the Jurassic floor is 65 m and the average aquifuge thickness of the working face is 140 m. In this area, the development of karst is high and aquifer is of high water yield property. Jurassic fissure water is the indirect water-filling source and the main water-bearing aquifer. The confined water pressure is about 1.81–3.82 MPa. There is a fault group that is inclined through the 13301 working face, which connects the Jurassic aquifer and the working face. The transmitting ability is high. Moreover, the fault largely outcropped in the 13301 working face, with highly developed fractures.

Table 6 The weight vector of the judgment matrix

Assessment index	Weight vector
W_B	[0.429, 0.618], [0.117, 0.120], [0.200, 0.266], [0.109, 0.142]
W_1	[0.227, 0.247], [0.412, 0.448], [0.311, 0.327]
W_2	[0.508, 0.610], [0.118, 0.156], [0.280, 0.311]
W_3	[0.192, 0.231], [0.415, 0.497], [0.291, 0.329]
W_4	[0.124, 0.129], [0.247, 0.296], [0.566, 0.593]

The interval fuzzy comprehensive evaluation indices of floor water inrush, based on the mine's geological and hydrogeological conditions, are presented in Table 4. The interval numbers used are based on the proposed computational model. The result vectors, weight vectors, and judgment matrixes are expressed by interval numbers. Based on the different forms of membership functions, the interval membership degrees of the different indices can be calculated using Eqs. (10)–(14). The Gaussian membership functions for qualitative and quantitative indices are shown in Table 3. The interval membership degrees are shown in Table 5. According to the 1–9 scale AHP method, the interval judgment matrixes V_i can be obtained as follows.

$$V_A = \begin{bmatrix} [1,1] & [2,5] & [2,7] & [3,5] \\ [1/5,1/2] & [1,1] & [1/3,1/2] & [1,1] \\ [1/7,1/2] & [2,3] & [2,3] & [2,4] \\ [1/5,1/3] & [1/3,1/2] & [1/4,1/2] & [1,1] \end{bmatrix} \tag{24}$$

Table 7 The results of floor water inrush

Sample	Weight vector	Risk grade	Evaluation value	Actual value
Sample A	[[0.258, 0.421], [0.290, 0.582], [0.181, 0.230], [0.095, 0.141], [0.078, 0.104]]	II	1	1
Sample B	[[0.180, 0.593], [0.255, 0.624], [0.226, 0.472], [0.105, 0.288], [0.080, 0.171]]	II	1	1
Sample C	[[0.198, 0.251], [0.258, 0.323], [0.259, 0.389], [0.171, 0.260], [0.072, 0.135]]	III	0	0
Sample D	[[0.192, 0.396], [0.300, 0.522], [0.274, 0.459], [0.103, 0.161], [0.109, 0.148]]	II	1	1

Sample A: 3612 working face in Huaibei Permian mining area

Sample B: 3137 working face in Zhaozhuang Mine

Sample C: 7304 working face in Taoyang Mine

Sample D: 31401 working face in Bulianta Mine

$$V_{B1} = \begin{bmatrix} [1,1] & [1/3,1/2] & [1,1] \\ [2,3] & [1,1] & [1,1] \\ [1,1] & [1,1] & [1,1] \end{bmatrix} \quad (25)$$

$$V_{B2} = \begin{bmatrix} [1,1] & [2,5] & [2,3] \\ [1/5,1/2] & [1,1] & [1/4,1/2] \\ [1/3,1/2] & [2,4] & [1,1] \end{bmatrix} \quad (26)$$

$$V_{B3} = \begin{bmatrix} [1,1] & [1/5,1/2] & [1,1] \\ [2,5] & [1,1] & [1,1] \\ [1,1] & [1,1] & [1,1] \end{bmatrix} \quad (27)$$

$$V_{B4} = \begin{bmatrix} [1,1] & [1/4,1/2] & [1/4,1/3] \\ [2,4] & [1,1] & [1/4,1/2] \\ [3,4] & [2,4] & [1,1] \end{bmatrix}. \quad (28)$$

Then, the weight vectors W_B and W_i are achieved by Eqs. (16)–(21), as shown in Table 6. W_B and W_i are the weights of the criteria layer (B) and the indicator layer (C), respectively. Combining the determined judgment matrix N_C and the weight vectors W_B and W_i, the final interval number result vector can be obtained as follows:

$$N_A = [[0.119, 0.605], [0.155, 0.821], [0.042, 0.458], [0.004, 0.248], [0.029, 0.149]] \quad (29)$$

Based on the proposed relative superiority analysis method, the relative superiority of the final result vector can was calculated using Eqs. (22) and (23). Finally, the final evaluation grade of floor water inrush was determined:

$$P = [0.248,\ 0.275,\ 0.209,\ 0.115,\ 0.124]. \quad (30)$$

Based on the evaluation result and maximum weight principle, the final level of the 13301 working face is II, making it a high risk mining face. In fact, the working face was put into production on April 15, 2012, and floor water inrush often occurs. Moreover, the water quantity is gradually increasing. The maximum water inflow is about 800 m^3/h, which is in accordance with this evaluation result.

More Engineering Applications

To verify the rationality and operability of the proposed method, more case information was collected (Table 4; Sun 2014; Wang et al. 2012; Xiao et al. 2015; Yi et al. 2008). The results are presented as Table 7. Assume 0 for no occurrence of floor water inrush and 1 for its occurrence (Wang et al. 2012). The final results, based on the proposed method, demonstrates a high degree of consistency with the practical situation. The accuracy and reliability of the proposed method was assessed by comparison. Therefore, this methodology, based on the nonlinear fuzzy method and AHP, have practical g significance to mining, tunneling, and other engineering construction projects.

Conclusions

A new assessment method of floor water inrush in coal mines was established in this study, consisting of a multi-index evaluation system and a nonlinear fuzzy interval assessment model.

1. The risk level of floor water inrush was divided into five grades and the quantitative standards of the indexes were determined. Specifically, fault density, fault -water transmitting ability, the development degree of fracture and fault, hydraulic pressure of the confined aquifer, water supply condition and aquifer water yield property, karst development, floor aquifuge thickness, floor aquifuge strength, key aquifuge of floor, mining thickness, mining depth and inclined length of the mining face were taken into account.

2. The nonlinear fuzzy interval assessment model was established based on the established multi-index evaluation system, nonlinear fuzzy mathematics and AHP. The final evaluation level was calculated by the relative superiority analysis method.
3. The new established assessment method was successfully applied to the 13301 working face of the Wanglou mine. The results obtained by the proposed method agreed well with the practical situation, which verified the reliability of this approach.

Acknowledgements Much of the work presented in this paper was supported by the National Basic Research Program of China (973 Program, Grant 2013CB036002), the National Natural Science Foundation of China (Grant 51509147), and the promotive research fund for excellent young and middle-aged scientists of Shandong Province (Grant BS2014NJ004). The authors thank the reviewers for their valuable comments and suggestions that helped improve the quality of the paper.

References

Gao L (2014) Analysis on water bursting source and channel at 13301 working face of Wang Lou coal mine. Shandong Coal Sci Technol 8:162–166 **(in Chinese)**

Li B, Chen YL (2016) Risk assessment of coal floor water inrush from underlying aquifers based on GRA–AHP and its application. Geotech Geol Eng 34(1):143–154

Li LP, Zhou ZQ, Li SC, Xue YG, Xu ZH, Shi SS (2015) An attribute synthetic evaluation system for risk assessment of floor water inrush in coal mines. Mine Water Environ 34(3):288–294

Meng ZP, Li GQ, Xie XT (2012) A geological assessment method of floor water inrush risk and its application. Eng Geol 143–144(4):51–60

Moore R, Lodwick W (2003) Interval analysis and fuzzy set theory. Fuzzy Sets Syst 135(1):5–9

Qiao W, Li WP, Li T, Chang JY, Wang QQ (2017) Effects of coal mining on shallow water resources in semiarid regions: a case study in the Shennan mining area, Shaanxi, China. Mine Water Environ 36(1):104–113

Qiao W, Li WP, Li T, Zhang X, Wang YZ, Chen YK (2018) Relevance between hydrochemical and hydrodynamic data in a deep karstified limestone aquifer: a mining area case study. Mine Water Environ 37(2):393–404

Qiu M, Shi LQ, Teng C, Zhou Y (2017) Assessment of water inrush risk using the fuzzy delphi analytic hierarchy process and grey relational analysis in the Liangzhuang coal mine, China. Mine Water Environ 36(1):39–50

Saaty TL (1990) How to make a decision: The analytic hierarchy process. Eur J Oper Res 48(1):9–26

Shi L, Qiu M, Wei W, Xu D, Han J (2014) Water inrush evaluation of coal seam floor by integrating the water inrush coefficient and the information of water abundance. Int J Min Sci Technol 24(5):677–681 **(in Chinese)**

Shi LQ, Gao WF, Han J, Tan XP (2017) A nonlinear risk evaluation method for water inrush through the seam floor. Mine Water Environ 36(4):597–605

Sun K (2014) Evaluation of the water inrush risk of the no.7 coal seam floor in Taoyang coal mine. Shandong U Sci Technol PhD diss **(in Chinese)**

Sun JM, Wu J (2014) Optimization study on interval number judgment matrix weight vector based on immune evolution algorithm. Int J Secur Appl 8(3):355–362

Wang Y, Yang WF, Li M, Liu X (2012) Risk assessment of floor water inrush in coal mines based on secondary fuzzy comprehensive evaluation. Int J Rock Mech Min Sci 52(6):50–55

Wang X, Wang T, Wang Q, Liu X, Li R, Liu BJ (2017) Evaluation of floor water inrush based on fractal theory and an improved analytic hierarchy process. Mine Water Environ 36(1):87–95

Wang XT, Li SC, Xu ZH, Hu J, Pan DD, Xue YG (2018) Risk assessment of water inrush in karst tunnels excavation based on normal cloud model. Bull Eng Geol Environ. https://doi.org/10.1007/s1006401812946

Wu Q, Liu YZ, Liu DH, Zhou WF (2011) Prediction of floor water inrush: the application of GIS-based AHP vulnerable index method to Donghuantuo coal mine, China. Rock Mech Rock Eng 44(5):591–600

Wu Q, Zhaoc DK, Wang Y, Shen JK, Mu WP, Liu HL (2017) Method for assessing coal-floor water-inrush risk based on the variable-weight model and unascertained measure theory. Hydrogeol J 25(7):2089–2103

Xiao QH (2014) Study on water treatment technology of complex aquifer in 13301 working face of Wanglou Coal Mine. Shandong Coal Sci Technol 10:155–157 **(in Chinese)**

Xiao XF, Liu ZS, Jia YY (2015) Prediction and prevention technology of dynamic phenomena in Zhaozhuang mine in Kailua. Coal Mine Support 4:29–32 **(in Chinese)**

Yang BB, Sui WH, Duan LH (2017) Risk assessment of water inrush in an underground coal mine based on GIS and fuzzy set theory. Mine Water Environ 36(4):617–627

Yi MS, Zhu WB, Li L, Zhao X, Xu JL (2008) Water inrush mechanism and prevention for fourth panel roof in Bulianta coal mine. J China Coal Soc 33(3):241–245 **(in Chinese)**